Non-Linear Bit Arrangements in Genetic Algorithms

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Abstract

Our earlier research laid a theoretical basis for the contention that genetic algorithms can succeed even if bits are arranged in configurations different from a linear sequence. In the current research we show this success happens in practice, for a number of examples.

In (Greene, 2000) we extended Holland’s classic Schema Theorem to the setting that bits are arranged, not necessarily in a linear sequence, but at the nodes of a connected graph. For such potentially much liberalized ways of arranging bits (and given reasonable assumptions; see our theorem), one can expect schema of above average fitness to flourish.

In the current research, we put theory aside and see what happens in practice. We consider a dozen examples, of several different types, wherein bits are arranged in alternative geometries. Of course, with each problem and its associated bit geometry, we should use a crossover operator that is compatible with the geometry, and also conducive to the accumulation of desirable chromosomal subparts. This, coupled with the other familiar forces of geneticism, does lead, in our experiments, to convergence to individuals of high or even maximum fitness.

In this abstract we outline several of our examples.

(1) Twenty Queens Problem. This is the extension of the familiar Eight Queens Problem, to a chess board whose edge-size is 20 squares. A population individual is a $20 \times 20$ grid of bits, with 0, resp., 1, meaning the square is empty, resp., is occupied by a queen. For crossover, a random row or column or diagonal line is chosen, then a child is formed by copying bits from one side of the line in one parent and from the other side of the line in the other parent. The initial population consists of individuals each of whose $20 \times 20 = 400$ bits are chosen randomly; note such an individual starts with far too many queens. An individual’s error is found by summing the number of inappropriate queens situated on each row and column and diagonal; maximum error = 1482. An individual’s fitness is defined as maximum error minus own error; maximum fitness = 1482, too. Note that the fitness landscape is complex, with many global maxima and many local maxima. Many constraints must be satisfied by a solution to this problem. Experimental results: On 20 trials, each stopped after 2000 generations (population size = 100), the best individual encountered had an average fitness of 1472.1, or 99.3% of the maximum fitness.

(2) A population individual is a complete binary tree of 511 bits. There are 256 leaves on the tree’s bottom level, which is at depth 8. For crossover, at random we choose one of the 510 proper subtrees, then form a child by replacing that subtree in one parent by that of the other parent. We define the fitness of an individual to be the number of sibling nodes which have the same bit value (whether it be 0 or 1). There are 255 sibling pairs, so maximum fitness is 255. Note that the fitness landscape has many global and many local maxima. Experimental results: On each of 20 trials, an individual of maximum fitness was found, upon average generation number 480.45.

(3) Bits are arranged in a 3-dimensional cube. Specifically, the bits are positioned at the points $(j, k, l)$ of Euclidean 3-space for which each of $j, k, l$ is an integer in the range 0..10. There are $11^3 = 1331$ bits altogether. For crossover, we cut a cube with a random plane, then form a child by copying bits from one side of that plane in one parent and from the other side of that plane in the other parent. Define the point antipodal to $(j, k, l)$ to be the one located symmetrically across the center of our cube; thus, it has coordinates $(10-j, 10-k, 10-l)$. An individual’s fitness we take to be the number of antipodal pairs in it which have the same bit value (whether 0 or 1); maximum fitness = 665. Experimental results: On 20 trials, each running to at most 2000 generations, the best individual found had an average fitness of 660.4, or 99.3% of the maximum fitness.

Reference